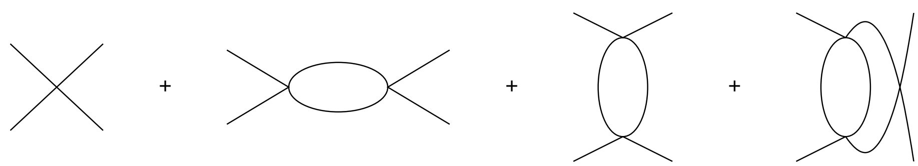
Physics G8069
homework #7
due Mon. Oct. 30

1. **Renormalization and scattering**

   (i) Write down the one-loop four-point scattering amplitude in $\frac{1}{4!}\lambda\phi^4$ theory, coming from the diagrams

   To regulate the diagrams you should Wick rotate to Euclidean space and put a cutoff on the magnitude of the Euclidean momentum: $k^2_{\text{Euclidean}} < \Lambda^2$. (You don't need to evaluate any loop integrals.)

   (ii) It's useful to reorganize perturbation theory as an expansion in the renormalized coupling $\lambda(\mu)$, defined by
$$\frac{1}{\lambda(\Lambda)} = \frac{1}{\lambda(\mu)} - \frac{3}{16\pi^2}\log\frac{\Lambda}{\mu}\,.$$
   Rewrite your scattering amplitude as an expansion in powers of $\lambda(\mu)$ up to $\mathcal{O}(\lambda(\mu)^2)$.

   (iii) You can "improve" perturbation theory by choosing μ in order to make the $\mathcal{O}(\lambda(\mu)^2)$ terms in your scattering amplitude as small as possible. Suppose you were interested in soft scattering, $s \approx t \approx u \approx 0$. What value of μ should you use? Alternatively, suppose you were interested in the "deep Euclidean" regime where s, t, u are large and negative (meaning $s \approx t \approx u \ll -m^2$). Now what value of μ should you use? (Here s, t, u are the usual Mandelstam variables. The values I'm suggesting do not satisfy the mass-shell condition $s + t + u = 4m^2$; if this bothers you imagine embedding the four-point amplitude inside a larger diagram.)

Moral of the story: it's best to work in terms of a renormalized coupling evaluated at the energy scale relevant to the process you're considering.