Physics G8070
homework #2
due Weds. Feb. 7

1. **Component expansion of the action**

   Work out the expansion of the action

   $$S = \int dt d^2\theta \left(-\frac{1}{2}\bar{D}FDF - W(F) \right)$$

   in terms of the component fields x, ψ, $\bar{\psi}$, d.

2. **Corrections to the ground state energy**

   Consider supersymmetric quantum mechanics with superpotential

   $$W(x) = \frac{1}{2}\omega x^2 + \frac{1}{3}gx^3 \,.$$

   When $g = 0$ this reduces to a supersymmetric harmonic oscillator, which has a unique zero-energy ground state that's invariant under supersymmetry. Use conventional perturbation theory to compute the $\mathcal{O}(g)$ and $\mathcal{O}(g^2)$ corrections to the energy of the ground state.

   Assuming these corrections vanish, you've found evidence for a "perturbative non-renormalization theorem:" the energy of the ground state isn't corrected at any finite order in perturbation theory. However as we showed in class non-perturbative effects break supersymmetry and lift the energy of the ground state.

3. **Majorana spinors**

   (i) Charge conjugation for a Dirac spinor ψ_D is defined by $\psi_D^c = -i\gamma^2\psi_D^*$. How does charge conjugation act in 2-component language, where we set $\psi_D = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$?

   (ii) A Majorana spinor ψ_M is a Dirac spinor that satisfies $\psi_M = \psi_M^c$. What does this imply for the 2-component spinors that make up ψ_M?

(iii) The Lagrangian for a Majorana spinor with mass m is

$$\mathcal{L} = -\frac{i}{2}\bar{\psi}_M \gamma^\mu \partial_\mu \psi_M - \frac{1}{2} m \bar{\psi}_M \psi_M$$

(the overbars denote the Dirac adjoint, not complex conjugation). Translate this Lagrangian into 2-component language.