Physics G8069
homework #2
due Mon. Sept. 25

1. **Flavor wavefunctions for the baryon octet**

   The baryon octet can be represented as a 3-index tensor $B^{ijk} = T^i{}_l \epsilon^{ljk}$ where $T^i{}_l$ is traceless. For example, in a basis $u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, the matrix $T = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ gives the flavor wavefunction of a proton $u(ud - du)$. Work out the flavor wavefunctions of the remaining members of the baryon octet. The hard part is getting the Σ^0 and Λ right; you'll need to take linear combinations which have the right isospin.

2. **Combining flavor + spin wavefunctions for the baryon octet**

   You might object to the octet wavefunctions worked out in problem 1 on the grounds that they don't respect Fermi statistics. For spin-1/2 baryons we can represent the spin of the baryon using a vector v^a $a = 1, 2$ which transforms in the **2** of the $SU(2)$ angular momentum group.

   (i) Write down a 3-index tensor that gives the spin wavefunction for the (spin-1/2) quarks that make up the baryon. (v^a is the analog of $T^i{}_l$ in problem 1. I'm asking you to find the analog of B^{ijk}.)

   (ii) Show how to combine your flavor and spin wavefunctions to make a state that is totally *symmetric* under exchange of any two quarks. It has to be totally symmetric so that, when combined with a totally antisymmetric color wavefunction, we get something that respects Fermi statistics.

   (iii) Suppose the quarks have no orbital angular momentum (as is usually the case in the ground state). Can you make an octet of baryons with spin 3/2?

3. **Electromagnetic decays of the Σ^***

The up and down quarks have different electric charges, so electromagnetic interactions violate the isospin $SU(2)$ subgroup of $SU(3)$. However the down and strange quarks have identical electric charges. This means that electromagnetism respects a *different* $SU(2)$ subgroup of $SU(3)$, that acts on the quarks as

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & U \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix} .$$

Use this to show that the electromagnetic decay $\Sigma^{*-} \rightarrow \Sigma^- \gamma$ is forbidden but that $\Sigma^{*+} \rightarrow \Sigma^+ \gamma$ is allowed.

4. **Decays of the W and τ**

The W^- boson decays to a "weak doublet" pair of fermions: either $(e^- \, \bar{\nu}_e)$, $(\mu^- \, \bar{\nu}_\mu)$, $(\tau^- \, \bar{\nu}_\tau)$, $(\bar{u} \, d)$, $(\bar{c} \, s)$, or (in principle) $(\bar{t} \, b)$.

(i) Suppose the amplitude for W^- decay is the same for all fermion pairs. Only kinematically allowed decays are possible, but aside from that you can neglect differences in phase space due to fermion masses. What would you predict for the branching ratios

$$\begin{array}{l} \Gamma(W^- \rightarrow e^- \bar{\nu}_e) \\ \Gamma(W^- \rightarrow \mu^- \bar{\nu}_\mu) \\ \Gamma(W^- \rightarrow \tau^- \bar{\nu}_\tau) \\ \Gamma(W^- \rightarrow \text{hadrons}) \end{array}$$

(ii) The τ^- lepton decays via $\tau^- \rightarrow W^- \nu_\tau$ followed by W^- decay. What would you predict for the branching ratios

$$\begin{array}{l} \Gamma(\tau^- \rightarrow e^- \bar{\nu}_e) \\ \Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu) \\ \Gamma(\tau^- \rightarrow \text{hadrons}) \end{array}$$

(iii) How did you do, compared to the particle data book? What would happen if you didn't take color into account?