Physics G8069
Fall 2006
Final exam

You can use whatever references you like (class notes, books,...) but please do your own work. The exam is due Thursday 12/14 at 5pm. You can give it to me in my office or leave it in my mailbox on the 7th floor. Come see me if you have any questions. Good luck!

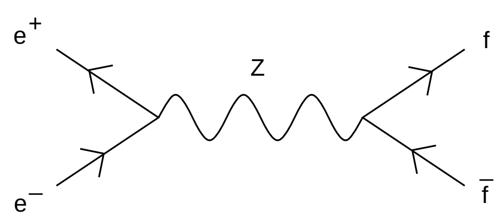
1. **Polarization asymmetry at the Z pole**

   SLAC studied $e^- e^+ \to f\bar{f}$ at the Z pole with a polarized e^- beam. The polarization asymmetry is defined by

   $$A_{LR} = \frac{\sigma(e^-_L e^+_R \to f\bar{f}) - \sigma(e^-_R e^+_L \to f\bar{f})}{\sigma(e^-_L e^+_R \to f\bar{f}) + \sigma(e^-_R e^+_L \to f\bar{f})}$$

   where the subscripts indicate the helicity of the particles. For simplicity let's neglect the mass of the electron.

   (i) Write down the amplitude for the basic process

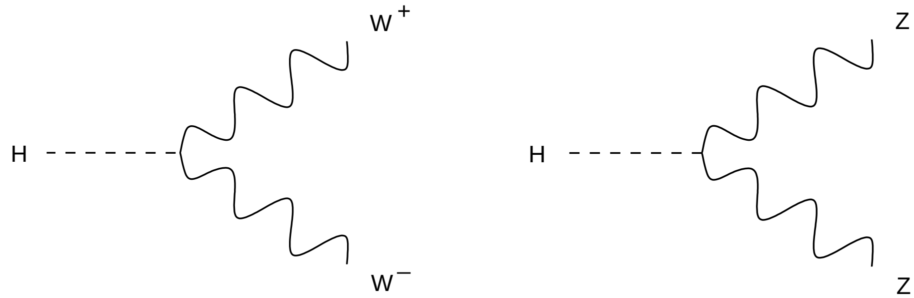
   (ii) Write down the amplitude when the incoming electron is polarized, either left-handed $\gamma^5 u(p_1) = -u(p_1)$ or right-handed $\gamma^5 u(p_1) = +u(p_1)$.

   (iii) Compute A_{LR} in terms of $\sin^2\theta_W$. No trace theorems are required!

   (iv) The observed asymmetry in $e^- e^+ \to$ hadrons is $A_{LR} = 0.1514 \pm 0.002$. How well did you do?

2. $H \to W^+W^-$ **and** $H \to ZZ$

   Compute the partial widths Γ_{WW} and Γ_{ZZ} for the decays $H \to W^+W^-$ and $H \to ZZ$ from the diagrams

   The Feynman rules can be found on p. 511 of Cheng & Li (where H is denoted ϕ_1) as well as on p. 131 of Quigg. For large Higgs mass show that the decay is predominantly to longitudinally-polarized vector bosons: that is, show that for $m_H \gg m_W$ the partial width for $H \to W^+W^-$ is dominated by $H \to W_L^+W_L^-$ and likewise for $H \to ZZ$.

3. **QCD and the** W **and** Z **masses**

   Consider a theory which resembles the standard model in every respect except that it doesn't have a Higgs field. You can get the Lagrangian for this theory by setting $\phi = 0$ in the standard model Lagrangian; the Dirac and Yang-Mills terms survive while the Higgs and Yukawa terms drop out. In such a theory, what are the masses of the W and Z bosons?

   Before your answer "zero," recall the effective Lagrangian for chiral symmetry breaking by the strong interactions, $\mathcal{L} = \frac{1}{4} f^2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)$. Let's concentrate on the up and down quarks so that $U \in SU(2)$.

   (i) By introducing suitable covariant derivatives, write the effective Lagrangian which describes the couplings between U and the $SU(2)_L \times U(1)_Y$ gauge fields W_μ, B_μ.

   (ii) Compute the W and Z masses in terms of f and the $SU(2)_L \times U(1)_Y$ gauge couplings g, g'.

   (iii) In this model, what particles get eaten to give the W and Z bosons a mass?

   (iv) In the real world, taking both the Higgs field and QCD effects into account, how much does chiral symmetry breaking by the strong interactions contribute to the masses of the W and Z bosons?