superfield		$SU(3)_C \times SU(2)_L \times U(1)_Y$	particle content	R-parity
vector	V_3	$(\mathbf{8}, \mathbf{1}, 0)$	gluons g and gluinos $\tilde{g}$	$+$
	V_2	$(\mathbf{1}, \mathbf{3}, 0)$	W-bosons W and winos $\tilde{W}$	$+$
	V_1	$(\mathbf{1}, \mathbf{1}, 0)$	hypercharge boson B and bino $\tilde{B}$	$+$
chiral	L_i	$(\mathbf{1}, \mathbf{2}, -1)$	left-handed leptons $\binom{\nu}{e}_{Li}$ and sleptons $\binom{\tilde{\nu}}{\tilde{e}}_{Li}$	$-$
	$\bar{e}_i$	$(\mathbf{1}, \mathbf{1}, 2)$	right-handed leptons $\bar{e}_{Ri}$ and sleptons $\tilde{e}^*_{Ri}$	$-$
	Q_i	$(\mathbf{3}, \mathbf{2}, 1/3)$	left-handed quarks $\binom{u}{d}_{Li}$ and squarks $\binom{\tilde{u}}{\tilde{d}}_{Li}$	$-$
	$\bar{u}_i$	$(\bar{\mathbf{3}}, \mathbf{1}, -4/3)$	right-handed quarks $\bar{u}_{Ri}$ and squarks $\tilde{u}^*_{Ri}$	$-$
	$\bar{d}_i$	$(\bar{\mathbf{3}}, \mathbf{1}, 2/3)$	right-handed quarks $\bar{d}_{Ri}$ and squarks $\tilde{d}^*_{Ri}$	$-$
	H_u	$(\mathbf{1}, \mathbf{2}, 1)$	up-type Higgs $\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$ and Higgsinos $\begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$	$+$
	H_d	$(\mathbf{1}, \mathbf{2}, -1)$	down-type Higgs $\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ and Higgsinos $\begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$	$+$

Notation: $i = 1, 2, 3$ is a generation index. The superfields $\bar{e}_i$, $\bar{u}_i$, $\bar{d}_i$ are chiral (not anti-chiral) – the overbar is part of their name, it doesn't indicate complex conjugation. The component fields $\bar{e}_R$, $\bar{u}_R$, $\bar{d}_R$ are left-handed chiral spinors – the overbar indicates that they're related by complex conjugation to the right-handed components of a Dirac spinor. The squarks and sleptons are all complex scalar fields; rather than indicating helicity the subscripts L, R tell which fermion they're related to by supersymmetry.

Besides canonical kinetic terms for all superfields, the MSSM is taken to have the most general superpotential invariant under R-parity, namely

$$W_{MSSM} = \mu H_u H_d + \lambda_u^{ij} Q_i \bar{u}_j H_u + \lambda_d^{ij} Q_i \bar{d}_j H_d + \lambda_e^{ij} L_i \bar{e}_j H_d$$

I've suppressed all gauge indices. There's a unique way of contracting them (with two-index ϵ-tensors if need be) to make the superpotential gauge invariant.

Soft terms

We add the following soft susy-breaking terms to the MSSM Lagrangian.

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} &= (m^2_{\tilde{q}})^{ij}\tilde{q}_i^\dagger\tilde{q}_j + (m^2_{\tilde{u}})^{ij}\tilde{u}_i^\dagger\tilde{u}_j + (m^2_{\tilde{d}})^{ij}\tilde{d}_i^\dagger\tilde{d}_j \\
&+ (m^2_{\tilde{l}})^{ij}\tilde{l}_i^\dagger\tilde{l}_j + (m^2_{\tilde{e}})^{ij}\tilde{e}_i^\dagger\tilde{e}_j \\
&+ m^2_{h_u} h_u^\dagger h_u + m^2_{h_d} h_d^\dagger h_d \\
&+ (B\mu h_u h_d + \text{c.c.}) \\
&+ \left(2m_3 \text{Tr}(\tilde{g}\tilde{g}) + 2m_2 \text{Tr}(\tilde{W}\tilde{W}) + m_1 \tilde{B}\tilde{B} + \text{c.c.}\right) \\
&+ \left(A_u^{ij}\tilde{q}_i h_u \tilde{u}_j^* + A_d^{ij}\tilde{q}_i h_d \tilde{d}_j^* + A_e^{ij}\tilde{l}_i h_d \tilde{e}_j^* + \text{c.c.}\right) \\
&+ \left(C_u^{ij}\tilde{q}_i h_d^* \tilde{u}_j^* + C_d^{ij}\tilde{q}_i h_u^* \tilde{d}_j^* + C_e^{ij}\tilde{l}_i h_u^* \tilde{e}_j^* + \text{c.c.}\right)
\end{aligned}$$

Notation: $\tilde{q}_i = \binom{\tilde{u}}{\tilde{d}}_{Li}$ are the left-handed squarks, $\tilde{u}_i = \tilde{u}_{Ri}$ and $\tilde{d}_i = \tilde{d}_{Ri}$ are the right-handed squarks, $\tilde{l}_i = \binom{\tilde{\nu}}{\tilde{e}}_{Li}$ are the left-handed sleptons, $\tilde{e}_i = \tilde{e}_{Ri}$ are the right-handed sleptons, $h_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$ and $h_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$ are the Higgs doublets. Remember it's the complex conjugates of $\tilde{u}_i$, $\tilde{d}_i$, $\tilde{e}_i$ that sit in chiral superfields. For the most part people neglect the C terms.

Higgs spectrum

After electroweak symmetry breaking we have

$$\langle h_u \rangle = \begin{pmatrix} 0 \\ v_u/\sqrt{2} \end{pmatrix} \qquad \langle h_d \rangle = \begin{pmatrix} v_d/\sqrt{2} \\ 0 \end{pmatrix}$$

The ratio of Higgs vevs is denoted $\tan\beta = v_u/v_d$, while $v = \sqrt{v_u^2 + v_d^2}$ replaces the standard model Higgs vev in gauge boson masses. To expand about this vacuum we set

$$h_u = \begin{pmatrix} H^+\cos\beta \\ \frac{1}{\sqrt{2}}(v_u + \hat{H}_u + iA^0\cos\beta) \end{pmatrix} \qquad h_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + \hat{H}_d + iA^0\sin\beta \\ H^-\sin\beta \end{pmatrix}$$

where $\hat{H}_u$, $\hat{H}_d$, A^0 are real and $H^+ = (H^-)^*$ is complex.

The neutral real Higgs scalar A^0 aquires a mass

$$m^2_{A^0} = 2B\mu/\sin 2\beta\,.$$

The fields $\hat{H}_u$, $\hat{H}_d$ mix to form light and heavy neutral Higgs scalars h^0, H^0.

$$m^2_{h^0,H^0} = \frac{1}{2}\left(m^2_{A^0} + m^2_Z \pm \sqrt{(m^2_{A^0} + m^2_Z)^2 - 4m^2_{A^0} m^2_Z \cos^2 2\beta}\right)$$

Finally the charged Higgs fields have a mass

$$m^2_{H^\pm} = m^2_{A^0} + m^2_W \,.$$

Standard model particle spectrum

The W and Z bosons couple to both Higgs doublets, so their masses are

$$\begin{aligned} m^2_W &= \frac{1}{4} g_2^2 \left(v_u^2 + v_d^2\right) = \frac{1}{4} g_2^2 v^2 \\ m^2_Z &= \frac{1}{4} \left(g_1^2 + g_2^2\right) \left(v_u^2 + v_d^2\right) = \frac{1}{4} g_Z^2 v^2 \end{aligned}$$

The up-type quarks get a mass from the H_u Higgs doublet, for example the top mass is

$$m_t = \frac{1}{\sqrt{2}} \lambda_t v_u \,.$$

The charged leptons and down-type quarks get a mass from the H_d Higgs doublet, for example the bottom mass is

$$m_b = \frac{1}{\sqrt{2}} \lambda_b v_d \,.$$

Neutralinos

The four neutral fermions $N = \left(\tilde{B}, \tilde{W}^0, \tilde{H_d}^0, \tilde{H_u}^0\right)^T$ mix to form neutralinos $\tilde{\chi}^0_i$, $i = 1, 2, 3, 4$. The mass matrix is

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} N^T M N + \text{c.c.} \\ M &= \begin{pmatrix} m_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & m_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix} \end{aligned}$$

Here $c_\beta = \cos\beta$, $c_W = \cos\theta_W$, etc.

Charginos

The charged fermions $C^+ = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u{}^+ \end{pmatrix}$, $C^- = \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d{}^- \end{pmatrix}$ mix to form charginos $\tilde{\chi}_i^\pm$, $i = 1, 2$. The mass matrix is (note that $C^+ \neq (C^-)^*$)

$$\mathcal{L} = -C^{-T} X C^+ + \text{c.c.}$$
$$X = \begin{pmatrix} m_2 & \sqrt{2} m_W s_\beta \\ \sqrt{2} m_W c_\beta & \mu \end{pmatrix}$$

You can diagonalize $X = U^T \begin{pmatrix} m_{\tilde{\chi}_1} & 0 \\ 0 & m_{\tilde{\chi}_2} \end{pmatrix} V$ using 2×2 unitary matrices U, V.

Gluinos

The gluinos can't mix with any other MSSM particles. They get a mass from the soft terms, $\mathcal{L} = -2m_3 \mathrm{Tr}(\tilde{g}\tilde{g}) + \text{c.c.}$

Squarks

Neglecting inter-generational mixing, the two top squarks $\tilde{t}_{L,R}$ will mix to form mass eigenstates $\tilde{t}_{1,2}$. Their mass matrix is

$$\mathcal{L} = -\begin{pmatrix} \tilde{t}_L^* & \tilde{t}_R^* \end{pmatrix} M^2 \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}$$
$$M^2 = \begin{pmatrix} m_{\tilde{q}_t}^2 + m_t^2 + \Delta_t & v_u A_t - \mu v_d \lambda_t \\ v_u A_t - \mu v_d \lambda_t & m_{\tilde{t}}^2 + m_t^2 + \Delta_{\bar{t}} \end{pmatrix}$$

where the D-term contributions are $\Delta_{t,\bar{t}} = (T_L^3 - Q_{\text{e.m.}} \sin^2\theta_W) m_Z^2 \cos 2\beta$. The mixing often makes $\tilde{t}_1$ the lightest squark.

For the remaining squarks the mixing is smaller. For the most part their masses arise directly from the mass terms in $\mathcal{L}_{\text{soft}}$.

Sleptons

Neglecting inter-generational mixing, the two tau sleptons $\tilde{\tau}_{L,R}$ will mix to form mass eigenstates $\tilde{\tau}_{1,2}$. Their mass matrix is

$$
\begin{aligned}
\mathcal{L} &= -\begin{pmatrix} \tilde{\tau}_L^* & \tilde{\tau}_R^* \end{pmatrix} M^2 \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix} \\
M^2 &= \begin{pmatrix} m_{\tilde{l}_\tau}^2 + m_\tau^2 + \Delta_\tau & v_d A_\tau - \mu v_u \lambda_\tau \\ v_d A_\tau - \mu v_u \lambda_\tau & m_{\tilde{\bar{\tau}}}^2 + m_\tau^2 + \Delta_{\bar{\tau}} \end{pmatrix}
\end{aligned}
$$

where again the D-term contributions are $\Delta_{\tau,\bar{\tau}} = (T_L^3 - Q_{\text{e.m.}} \sin^2\theta_W) m_Z^2 \cos 2\beta$. Due to mixing $\tilde{\tau}_1$ is often the lightest slepton.

For the remaining sleptons the mixing is smaller. For the most part their masses arise directly from the mass terms in $\mathcal{L}_{\text{soft}}$.