Physics G8070
homework #6
due Mon. April 9

1. **Polonyi superpotential**

   Consider a single chiral superfield Φ coupled to supergravity, with a canonical Kähler potential $K(\Phi, \Phi^*) = \Phi^* \Phi$ and a so-called Polonyi superpotential
   $$W(\Phi) = m^2(\Phi + 2 - \sqrt{3})\,.$$
   Here m^2 is a constant with units of $(\text{mass})^2$, and the peculiar $2 - \sqrt{3}$ is chosen for reasons you'll see below.

   (i) Compute the scalar potential $V(\phi, \phi^*)$.

   (ii) Show that the potential has a non-trivial minimum and find the vacuum expectation value of ϕ.

   (iii) Show that the vacuum energy vanishes; this was the reason for the peculiar constant.

   (iv) Show that supersymmetry is nonetheless spontaneously broken, and compute the gravitino mass $m_{3/2}$.

2. **No-scale supergravity**

   Consider a collection of chiral multiplets Φ, Y^i coupled to supergravity. Suppose the Kähler potential has the following "no-scale" form
   $$K(\Phi, \Phi^*, Y^i, Y^{i*}) = -3m_p^2 \log\left(f(\Phi) + \overline{f(\Phi)}\right) + \widetilde{K}(Y^i, Y^{i*})\,.$$
   Here $\widetilde{K}$ is an arbitrary real function. Also suppose the superpotential only depends on the fields Y^i, $W = W(Y^i)$.

   (i) Show that the scalar potential is given by
   $$V = \frac{1}{(f + \overline{f})^3} e^{\widetilde{K}/m_p^2} g^{i\bar{j}} D_i W \overline{D_j W}\,.$$
   Note that V can't be negative.

(ii) Suppose we can find expectation values for the fields y_i that make $D_i W = 0$. Then the scalar potential vanishes identically, leaving the vev of ϕ undetermined. Show however that supersymmetry is broken by computing the gravitino mass $m_{3/2}$.

Comments: this is an interesting model because it has vanishing vacuum energy and a flat direction parameterized by $\langle\phi\rangle$, even though susy is spontaneously broken.