Physics G8069
homework #1
due Mon. Sept. 18

1. **Decays of the spin–1/2 baryons**

   Most of the spin–1/2 baryons in the "baryon octet" (nucleon, Λ, Σ, Ξ) decay weakly, to another spin–1/2 baryon plus a pion. The two exceptions are the Σ^0 (which decays electromagnetically) and the neutron (which decays weakly to $pe^-\bar{\nu}_e$).

   - Show that none of the particles in the octet can decay strongly.
   - Show that the Σ^0 is the only particle in the octet that can decay electromagnetically.
   - Explain the unusual decay pattern for the neutron.

2. **Consequences of isospin**

   Suppose the strong interaction Hamiltonian is invariant under an $SU(2)$ isospin symmetry, $[H_{\text{strong}}, \mathbf{I}] = 0$. By inserting suitable isospin raising and lowering operators $I_\pm = I_1 \pm iI_2$ show that (up to possible phases)

$$\frac{1}{\sqrt{3}}\langle\Delta^{++}|H_{\text{strong}}|\pi^+ p\rangle = \frac{1}{\sqrt{2}}\langle\Delta^{+}|H_{\text{strong}}|\pi^0 p\rangle = \langle\Delta^{0}|H_{\text{strong}}|\pi^- p\rangle\,.$$

3. **Decay of the Ξ^***

   The Ξ^* baryon decays primarily to $\Xi + \pi$. For a neutral Ξ^* there are two possible decays:

$$\Xi^{*0} \to \Xi^0\pi^0$$

$$\Xi^{*0} \to \Xi^-\pi^+$$

   Use isospin to predict the branching ratios.

4. $\Delta I = 1/2$ **rule**

The Λ baryon decays weakly to a nucleon plus a pion. The Hamiltonian responsible for the decay is

$$H = \frac{1}{\sqrt{2}} G_F \bar{u}\gamma^\mu(1-\gamma^5)d\,\bar{s}\gamma_\mu(1-\gamma^5)u + \text{c.c.}$$

This operator changes the strangeness by ± 1 and the z component of isospin by $\mp 1/2$. It can be decomposed $H = H_{3/2} + H_{1/2}$ into pieces which carry total isospin 3/2 and 1/2, since $\bar{u}\gamma^\mu(1-\gamma^5)d$ transforms as $|1,-1\rangle$ and $\bar{s}\gamma_\mu(1-\gamma^5)u$ transforms as $|1/2, 1/2\rangle$. The (theoretically somewhat mysterious) "$\Delta I = 1/2$ rule" states that the $I = 1/2$ part of the Hamiltonian dominates.

(i) Use the $\Delta I = 1/2$ rule to relate the matrix elements $\langle p\,\pi^- | H | \Lambda\rangle$ and $\langle n\,\pi^0 | H | \Lambda\rangle$.

(ii) Predict the corresponding branching ratios for $\Lambda \to p\pi^-$ and $\Lambda \to n\pi^0$.

The PDG gives the branching ratios $\Lambda \to p\pi^- = 63.9\%$ and $\Lambda \to n\pi^0 = 35.8\%$.