Physics G8070
homework #1
due Mon. Jan. 29

1. **Equations of motion**

   (i) Derive the Euler-Lagrange equations of motion by varying the fields in
   $$L = \frac{1}{2}\dot{x}^2 + i\bar{\psi}\dot{\psi} - \frac{1}{2}(W')^2 + \bar{\psi}\psi W'' \,.$$
   Vary the fields in the usual way,
   $$x \to x + \delta x \qquad \psi \to \psi + \delta\psi \qquad \bar{\psi} \to \bar{\psi} + \delta\bar{\psi}\,,$$
   but remember that ψ, $\delta\psi$, $\bar{\psi}$, $\delta\bar{\psi}$ all anticommute.

   (ii) Use the (anti-) commutation relations
   $$i[p, x] = 1 \qquad \{\psi, \bar{\psi}\} = 1$$
   to derive the Heisenberg equations of motion for the Hamiltonian
   $$H = \frac{1}{2}p^2 + \frac{1}{2}(W')^2 - \frac{1}{2}(\bar{\psi}\psi - \psi\bar{\psi})W'' \,.$$
   Assuming your results match with part (i), this shows we quantized our fermions correctly.

2. **Supercharges from Noether's theorem**

   (i) Show that the Lagrangian
   $$L = \frac{1}{2}\dot{x}^2 + i\bar{\psi}\dot{\psi} - \frac{1}{2}(W')^2 + \bar{\psi}\psi W''$$
   changes by a total time derivative under
   $$\begin{aligned} \delta x &= \xi\psi + \bar{\psi}\bar{\xi} \\ \delta\psi &= -i\bar{\xi}(\dot{x} + iW') \\ \delta\bar{\psi} &= i\xi(\dot{x} - iW') \end{aligned}$$

   (ii) Use Noether's theorem to find the conserved charges Q, $\bar{Q}$ associated with this symmetry.

3. **Component expansion of the action**

   Work out the expansion of the action

$$S = \int dt d^2\theta \left(-\frac{1}{2} \bar{D} F D F - W(F) \right)$$

   in terms of the component fields x, ψ, $\bar{\psi}$, d.