Physics G8070
homework #3
due Mon. Feb. 19

1. ***R*-symmetry for the WZ model**

   Consider a transformation which multiplies the fermionic superspace coordinates by a phase while leaving the bosonic coordinates invariant.

   $$\theta_\alpha \to e^{-i\chi}\theta_\alpha \qquad \bar{\theta}_{\dot{\alpha}} \to e^{i\chi}\bar{\theta}_{\dot{\alpha}} \qquad x^\mu \text{ invariant}$$

   Here χ is an arbitrary angle. Such transformations are known as R-symmetries. A superfield which transforms according to

   $$\Phi(x,\theta,\bar{\theta}) \to e^{iq\chi}\Phi(x, e^{-i\chi}\theta, e^{i\chi}\bar{\theta})$$

   is said to have R-charge q.

   (i) Show that the WZ action

   $$S = \int d^4x d^4\theta\, \Phi^*\Phi + \int d^4x d^2\theta\, W(\Phi) + \int d^4x d^2\bar{\theta}\, \overline{W(\Phi)}$$

   is invariant under $U(1)_R$ provided W carries R-charge 2.

   (ii) Suppose $W = \frac{1}{3}g\Phi^3$ where g is a coupling constant. The WZ action is invariant under $U(1)_R$ if Φ carries R-charge 2/3. How do the component fields in Φ transform under $U(1)_R$?

   (iii) What's the classical conserved quantity (the "R-charge") associated with R-symmetry?

2. **Mass sum rules**

   Consider a general Wess-Zumino model with a collection of chiral superfields Φ_i, a canonical Kähler potential $K = \sum_i \Phi_i^*\Phi_i$ and an arbitrary superpotential $W(\Phi)$. Prove that the bose and fermi mass matrices obey the sum rule $\mathrm{Tr}\, M^2_{\text{bose}} = \mathrm{Tr}\, M^2_{\text{fermi}}$. A few comments:

   - You should be sure to expand about a minimum of the potential (the fields don't need to vanish in the ground state).

- In taking the bose trace you should work in terms of real scalar fields. In taking the fermi trace the two helicities of a Weyl spinor should count separately.

Note that if susy is spontaneously broken the individual bose and fermi masses will differ; it's only the sum of the squares of their masses that has to agree. By the way, this sum rule can be violated by radiative corrections.

3. **Goldstone fermions**

   Consider a general Wess-Zumino model with a collection of chiral superfields Φ_i, a canonical Kähler potential $K = \sum_i \Phi_i^* \Phi_i$ and an arbitrary superpotential $W(\Phi)$. Suppose that supersymmetry is spontaneously broken, meaning that some auxiliary field F_i is non-zero in the ground state. Show that the fermion mass matrix has a zero eigenvalue.

   Moral of the story: there's a massless "Goldstone fermion" associated with spontaneous susy breaking.